AP Calculus AB
Stuff You Must Know

Algebra Stuff

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard Form: $Ax + By = C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Trig Values

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\dfrac{\sqrt{3}}{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	∞

Trig Stuff Identities

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(2x) = 2\cos^2 x - 1$

$\cos(2x) = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

$\sin(-x) = -\sin(x)$

$\cos(-x) = \cos(x)$

$\tan(-x) = -\tan(x)$

$\cot(-x) = -\cot(x)$

$\sec(-x) = \sec(x)$

$\csc(-x) = -\csc(x)$

Differential Calculus Formulas and Rules

$$\frac{d}{dx}(x)^n = nx^{n-1}$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(uv) = uv' + vu'$$
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$
$$\frac{d}{dx}\left(f(g(x))\right) = f'\left(g(x)\right)g'(x)$$
$$\frac{d}{dx}\left(e^x\right) = e^x$$
$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$
$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$
$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}(arc\cot x) = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}(arc\sec x) = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}(arccs\text{c}\, x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

Applications of the first and second derivative

Curve Sketching

- To find a critical value, set $f'(x) = 0$ or undefined
- Use a sign chart to determine if the function has a relative extrema. Make sure you write sentences summarizing the results.
- Use can also use the Second Derivative Test to verify extrema. Suppose that x_0 is a critical value. If $f''\left(x_0\right) < 0,$ then x_0 is the *x*-coordinate of the relative maximum. If $f''\left(x_0\right) > 0,$ then x_0 is the *x*-coordinate of the relative minimum.
- To find points of inflection, set $f''(x) = 0$ or undefined. Then, show that the sign of $f''(x)$ changes as *x* passes through that point.

Three Important Theorems

Intermediate Value Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b] and *y* is some value between $f(a)$ and $f(b)$, then there exists at least one value $x = c$ in the open interval (a, b) where $f(c) = y$.

In other words, a continuous function must pass through every *y*-value between $f(a)$ and $f(b)$,.

Mean Value Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b] AND is differentiable on an open interval (a, b), then there exists at least one value $x = c$ in the open interval (a, b) where $f'(c) = \frac{f(b) - f(a)}{b - a}$.

In other words, there is at least one point on a smooth curve where the tangent line can be drawn so that it is parallel to the secant line drawn through the endpoints of the interval.

Rolle's Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b] AND is differentiable on an open interval (a, b) AND $f(a) = f(b)$, , then there exists at least one value $x = c$ in the open interval (a, b) where $f'(c) = 0$.

In other words, if the endpoints of a differentiable function have the same *y*-coordinates, there is at least one point inside the interval where the slope of the tangent line is equal to zero. This is a special case of the

Integral Formulas

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c; \; n \neq -1$

$\int \frac{1}{x} \, dx = \ln x + c$

$\int e^x \, dx = e^x + c$

$\int a^x \, dx = \frac{a^x}{\ln a} + c$

$\int \sin x \, dx = -\cos x + c$

$\int \cos \, dx = \sin x + c$

$\int \tan x \, dx = -\ln|\cos x| + c$

$\int \cot \, dx = \ln|\sin x| + c$

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$\int \csc x \, dx = -\ln|\csc x + \cot x| + c$

$\int \sec^2 x \, dx = \tan x + c$

$\int \csc^2 x \, dx = -\cot x + c$

$\int \sec x \tan x \, dx = \sec x + c$

$\int \csc x \cot x \, dx = -\csc x + c$

$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + c$

$\int \frac{1}{1+x^2} \, dx = \arctan x + c$

$\int \frac{1}{x\sqrt{x^2-1}} \, dx = arc\sec x + c$

(Integration by parts)

Fundamental Theorem of Calculus – Part 1	Fundamental Theorem of Calculus – Part 2	Average Value Theorem
$\int_a^b f'(x) \, dx = f(b) - f(a)$	$\frac{d}{dx}\left(\int_a^x f(t) \, dt\right) = f(x)$	If a function $f(x)$ is continuous on the closed interval [a, b], then there exists some number $x_0 = c$ such that $f(c) = \frac{1}{b-a}\int_a^b f(x) \, dx$
Volume of a Solid of Revolution (disk method) $V = \pi\int_a^b \left((OR)^2 - (IR)^2\right) \, dx \; or \; dy$	**Volume of a Solid with a Known Cross-Section** $V = \int_a^b Area(x) \, dx$	
	Particle Motion Formulas	
$velocity = \frac{d}{dt}(position)$	$acceleration = \frac{d}{dt}(velocity)$	$displacement = \int_{t_1}^{t_2} v(t) \, dt$
$total\ distance = \int_{t_1}^{t_2} \lvert v(t) \rvert \, dt$	$Avg\ velocity = \frac{position_2 - position_1}{time_2 - time_1}$	Final Position = $x(t_1) + \int_{t_1}^{t_2} v(t) \, dt$