Ch 7: Similarity

7-1 Ratios and Proportions
7-2 Similar Polygons
7-3 Proving Triangles Similar

7-4 Similarity in Right Triangles
7-5 Proportions in Triangles

7-1 Ratios and Proportions:

Focused Learning Target: I will be able to write ratios and solve proportions

Vocabulary:
- Ratio
- Proportion
- Extended Proportion

- Cross-Product Property
- Scale Drawing
- Scale

Ratio: a comparison of two quantities. Can be written as “the ratio of a to b”, a:b or \( \frac{a}{b} \).

Example:
A photo that is 8 in wide and 5 1/3 in high is enlarged to a poster that is 2 ft wide and 1 1/3 ft high. What is the ratio of the width of the photo to the width of the poster?

\[
\frac{\text{width of photo}}{\text{width of poster}} = \frac{8 \text{ in}}{2 \text{ ft}} = \frac{8 \text{ in}}{24 \text{ in}} = \frac{8}{24} = \frac{1}{3}
\]

The ratio is 1:3 or \( \frac{1}{3} \)

Proportion: a statement that two ratios are equal. You can write a proportion in these forms: \( \frac{a}{b} = \frac{c}{d} \) and \( a : b = c : d \)

Extended Proportion: when 3 or more proportions are equal, you can write an extended proportion:

\[
\frac{9}{27} = \frac{3}{9} = \frac{1}{3}
\]

Property

<table>
<thead>
<tr>
<th>Properties of Proportions</th>
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</thead>
<tbody>
<tr>
<td>( \frac{a}{b} = \frac{c}{d} ) is equivalent to</td>
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<tr>
<td>( (1) ) ( ad = bc )</td>
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<tr>
<td>( (2) ) ( \frac{b}{a} = \frac{d}{c} )</td>
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<tr>
<td>( (3) ) ( \frac{a}{c} = \frac{b}{d} )</td>
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<tr>
<td>( (4) ) ( \frac{a+b}{b} = \frac{c+d}{d} )</td>
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</tbody>
</table>

Property (1) is the result of the Cross-Product Property. It is also stated as the product of the “extremes is equal to the product of the means”

Example:

I’ll do one: \( \frac{a}{b} = \frac{c}{d} \)

We’ll do one: \( \frac{x-4}{3} = \frac{x+5}{5} \)

You try: \( \frac{2x}{7} = \frac{20}{5} \)
Properties of Proportions:

I’ll do four: if \( \frac{x}{y} = \frac{5}{6} \), complete each statement:

\[
\begin{array}{cccc}
a) 6x = & b) \frac{y}{x} = & c) \frac{x}{5} = & d) \frac{x+y}{y} = \\
& & & \\
\end{array}
\]

We’ll do four: if \( \frac{x}{y} = \frac{3}{4} \), complete each statement:

\[
\begin{array}{cccc}
a) 4x = & b) \frac{x+y}{y} = & c) \frac{x}{x+y} = & d) \frac{x+3}{3} = \\
& & & \\
\end{array}
\]

You do four: if \( \frac{a}{b} = \frac{1}{3} \), complete each statement:

\[
\begin{array}{cccc}
a) \frac{a+b}{b} = & b) 3a = & c) \frac{a+1}{1} = & d) \frac{a}{a+b} = \\
& & & \\
\end{array}
\]

Solving for a Variable:

I’ll do one: \( \frac{x}{7} = \frac{3}{5} \)

We’ll do one together: \( \frac{x-4}{5} = \frac{x}{3} \)

You try: \( \frac{x+1}{3} = \frac{x+3}{4} \)

7-2 Similar Polygons:

Focused Learning Target: I will be able to
- Identify similar polygons
- Apply similar polygons

CA Standard(s):
Geo 4.0: Students Prove basic theorems involving similarity.

Vocabulary:
- Similar
- Similarity Ratio

Similar: two polygons are similar if they have the same shape. They may or may not be the same size. The angles will be congruent and the sides will be proportional. The ratio of the lengths of corresponding sides is the similarity ratio. Similarity is indicated by the symbol \( \triangle \). Example: \( \triangle ABC \cong \triangle RST \) means “triangle ABC is similar to triangle RST”.


Understanding Similarity

If \( ABCD \sim EFGH \), then:
\[
\angle E = \angle B, \quad m \angle B = \angle F
\]
and,
\[
\frac{AB}{EF} = \frac{BC}{EH}
\]

Determining Similarity

I’ll do one:

Determine whether the triangles are similar. If they are, write a similarity statement and give the similarity ratio.

Identify corresponding angles and sides.
Check to see if all angles are congruent, and all pairs of corresponding sides are proportional.

We’ll do one together:

Determine whether the quadrilaterals are similar. If they are, write a similarity statement and give the similarity ratio.
You Try:

Determine whether the quadrilaterals are similar. If they are, write a similarity statement and give the similarity ratio.

Identify corresponding angles and sides.
Check to see if all angles are congruent, and all pairs of corresponding sides are proportional.

Using Similar Figures
I’ll do one:

\[ LMNO \sim QRST \]
Find the value of \( x \).
Is it a side or an angle? (if they are angles, then they are congruent. If they are sides, then they are proportional.)

We’ll do one together:

The quadrilaterals are similar. Find the value of each variable.
You Try:

The quadrilaterals are similar. Find the value of each variable.

You try another:

\( \triangle WLJ \sim \triangle QBV \). Find the value of each variable.

Note the difference in method between the two you try problems. In the space below, explain the reason for the difference.
7-3 Proving Triangles Similar:

Focused Learning Target: I will be able to
- Use AA, SAS and SSS similarity statements
- Apply AA, SAS, and SSS similarity statements

Vocabulary:
- Indirect measurement

<table>
<thead>
<tr>
<th>CA Standard(s):</th>
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<tbody>
<tr>
<td>Geo 2.0: Students write geometric proofs.</td>
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<tr>
<td>Geo 4.0: Students Prove basic theorems involving similarity.</td>
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<tr>
<td>Geo 5.0: Students prove that triangles are similar.</td>
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Postulate 7-1  Angle-Angle Similarity (AA ~) Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

\[ \triangle TRS \sim \triangle PLM \]

Using AA ~ postulate
I’ll do one:

<table>
<thead>
<tr>
<th>Given: ( \angle R \cong \angle L; \angle T \cong \angle P )</th>
<th>Prove: ( \triangle RTS \sim \triangle LPM )</th>
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<tbody>
<tr>
<td>Statements:</td>
<td>Reasons:</td>
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We’ll do one together:

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<tr>
<th>Given: ( m\angle R = 45^\circ; m\angle L = 45^\circ )</th>
<th>Prove: ( \triangle RSW \sim \triangle VSB )</th>
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You Try one:

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<tr>
<th>Given: ( \angle R \cong \angle W; \angle x \cong \angle E )</th>
<th>Prove: ( \triangle XYY \cong \triangle ENW )</th>
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**Theorem 7-1**  
**Side-Angle-Side Similarity (SAS \( \sim \)) Theorem**

If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar.

Using **SAS \( \sim \)** Theorem

I’ll do one:

<table>
<thead>
<tr>
<th>Given: ( \angle A \cong \angle Q; \frac{AB}{QR} = \frac{AC}{QS} )</th>
<th>Prove: ( \triangle ABC \sim \triangle QRS )</th>
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We’ll do one together:

**Given:** $CQ = 12; QM = 8; CT = 21; TP = 14$

**Prove:** $\triangle QCT \cong \triangle MCP$

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**You Try:**

**Given:** $XM = 3; MR = 9; TM = 15; MQ = 5$

**Prove:** $\triangle XMQ \cong \triangle RMT$

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### Theorem 7-2  Side-Side-Side Similarity (SSS ~) Theorem

If the corresponding sides of two triangles are proportional, then the triangles are similar.

Using SSS ~ Theorem
I’ll do one:

<table>
<thead>
<tr>
<th>Given: ( \frac{AB}{QR} = \frac{BC}{RS} = \frac{AC}{QS} )</th>
<th>Prove: ( \triangle ABC \sim \triangle QRS )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statements:</td>
<td>Reasons:</td>
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<tr>
<td>1. ( \frac{AB}{QR} = \frac{BC}{RS} )</td>
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<td>2. ( \frac{BC}{RS} = \frac{AC}{QS} )</td>
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<td>3. ( \frac{AC}{QS} = \frac{AB}{QR} )</td>
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<tr>
<td>4. ( \frac{AB}{QR} = \frac{BC}{RS} = \frac{AC}{QS} )</td>
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<tr>
<td>5. ( \triangle ABC \sim \triangle QRS )</td>
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We’ll do one together:

<table>
<thead>
<tr>
<th>Given: ( AB = 4; RQ = 8; AC = 5; QS = 10; BC = 6; RS = 12 )</th>
<th>Prove: ( \triangle ABC \sim \triangle QRS )</th>
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</thead>
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<tr>
<td>Statements:</td>
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<td>1. ( \frac{AB}{RQ} = \frac{BC}{RS} )</td>
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<tr>
<td>2. ( \frac{BC}{RS} = \frac{AC}{QS} )</td>
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<td>3. ( \frac{AC}{QS} = \frac{AB}{RQ} )</td>
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<td>4. ( \frac{AB}{RQ} = \frac{BC}{RS} = \frac{AC}{QS} )</td>
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<td>5. ( \triangle ABC \sim \triangle QRS )</td>
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</table>
You try:

Given:
\(AB = 1; RQ = 3; AC = 2; QS = 6; BC = 3; RS = 9\)
Prove: \(\triangle ABC \sim \triangle QRS\)

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Finding lengths in Similar Triangles
I’ll do one:

Find the value of \(x\) in the figure below:

We’ll do one together:

\(\triangle ABC \sim \triangle EBD\), Find DE.

You try:

\(AB = 6; RQ = 24; AC = 2\), find QS
You try another:

Ollie is looking at a mirror on the ground. In the mirror, he can see the top of a flagpole. If Ollie is 6 ft tall, he is 10 ft from the mirror and the flagpole is 30 ft from the mirror, how high is the top of the flagpole? Label the diagram using all of the given information and use what you know about similar triangles to calculate the height of the flagpole.

5-4 Similarity in Right Triangles:

<table>
<thead>
<tr>
<th>Focused Learning Target: I will be able to</th>
<th>CA Standard(s):</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Find and use relationships in similar right triangles</td>
<td>Geo 4.0: Students Prove basic theorems involving similarity.</td>
</tr>
<tr>
<td></td>
<td>Geo 5.0: Students prove that triangles are similar.</td>
</tr>
</tbody>
</table>

Vocabulary:

Geometric mean: is the positive number x such that \( \frac{a}{x} = \frac{x}{b} \). Note: \( x = \sqrt{ab} \)

Example 1: Finding the Geometric Mean

I’ll do one:

Find the geometric mean of 3 and 12

We’ll try one together:

Find the geometric mean of 15 and 20.

You try one:

Find the geometric mean of 2 and 22
**Theorem 7-3**

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

**Corollary**

**Corollary 2 to Theorem 7-3**

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse.

**Corollary**

**Corollary 1 to Theorem 7-3**

The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

Example 2: Applying Corollaries 1 and 2

I’ll do one:

Solve for \(x\) and \(y\)

![Diagram](image1)

We’ll try one together:

Solve for \(x\) and \(y\)

![Diagram](image2)

You try one:

Solve \(x\) and \(y\)

![Diagram](image3)
**Closure/Exit Ticket:**

1. Draw a right triangle with legs 5 cm and 12 cm long.
2. Draw the altitude to the hypotenuse.
3. Draw and label the two smaller triangles so that the corresponding parts line up. (use the right angle to assist you)

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**a.** find the length of the hypotenuse

**b.** Find the length of the altitude to the hypotenuse

**c.** Find the length of the segments of the hypotenuse formed by the altitude

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**7-5 Proportions in Triangles:**

<table>
<thead>
<tr>
<th>Focused Learning Target: I will be able to</th>
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<tbody>
<tr>
<td>• Use the Side-Splitter Theorem</td>
</tr>
<tr>
<td>• Use the Triangle-Angle-Bisector Theorem</td>
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<td>Geo 4.0: Students prove basic theorems involving similarity.</td>
</tr>
<tr>
<td>Geo 5.0: Students prove that triangles are similar.</td>
</tr>
<tr>
<td>Geo 7.0: Students prove and use theorems involving the properties of parallel lines cut by a transversal.</td>
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**Theorem 7-4 Side-Splitter Theorem**

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.
Example 1: Using the Side-Splitter Theorem
I’ll do one:

Solve for x

\[
\begin{array}{c}
2x \\
9 \\
12
\end{array}
\]

We’ll try one together:

Solve for x

\[
\begin{array}{c}
x \\
6 \\
x + 1 \\
9
\end{array}
\]

You try one:

Solve for x

\[
\begin{array}{c}
16 \\
S \\
10 \\
V
\end{array}
\]

Example 2: Using the Corollary to Theorem 7-4
I’ll do one:

Solve for x and y.

\[
\begin{array}{c}
16.5 \\
15 \\
30
\end{array}
\]

We’ll try one together:

Solve for x

\[
\begin{array}{c}
\text{Corollary to Theorem 7-4}
\end{array}
\]

If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

\[
\frac{a}{b} = \frac{c}{d}
\]
You try one:

Solve for \( x \).

Example 3: Using the Triangle-Angle-Bisector Theorem

I’ll do one:

Find the value of \( y \)

We’ll try one together:

Find the value of \( x \)

You try one:

Find the value of \( x \)
Closure/Exit Ticket:

In \( \triangle ABC \), \( \overline{QT} \parallel \overline{BC} \) and \( \overline{AM} \) bisects \( \angle BAC \). Find \( x \) and \( y \).

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\[ Q \]
\[ 5 \]
\[ 6 \]
\[ T \]
\[ X \]
\[ 3 \]
\[ 4 \]
\[ M \]
\[ y \]
\[ B \]
\[ C \]